Gauge-independent Thermal β Function In Yang-Mills Theory

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Abstract

It is proposed to use the pinch technique (PT) to obtain the gauge-independent thermal β function in a hot Yang-Mills gas. Calculations of the thermal β function are performed at one-loop level in four different gauges, (i) the background field method with an arbitrary gauge, (ii) the Feynman gauge, (iii) the Coulomb gauge, and (iv) the temporal axial gauge, and they yield the same result in all four cases.

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It is important for the study of the quark-gluon plasma and/or the evolution of the early Universe to fully understand the behaviour of the effective coupling constant $\alpha_s(= g^2/4\pi)$ in QCD at high temperature. The running of α_s with the temperature T and the external momentum $\kappa(= |\vec{\mathbf{k}}|)$ is governed by the thermal β function β_T [1]. However, the previous calculations of β_T have exposed various problems [2], a serious one of which is that the results are gauge-fixing dependent [3].

The background field method (BFM) has been applied to the calculation of β_T at one-loop [2][4][5]. First introduced by DeWitt [6], BFM is a technique for quantizing gauge field theories while retaining explicit gauge invariance for the background fields. Since the Green's functions constructed by BFM manifestly maintain gauge invariance, they obey the naive QED-like Ward identities. As a result, the spatial part of the three-gluon vertex, for static and symmetric external momenta, is related to the transverse function $\Pi_T(T, k_0 = 0, \kappa = |\vec{\mathbf{k}}|)$ of the polarization tensor $\Pi_{\mu\nu}$, and thus β_T is obtained in BFM from [2]

$$\beta_T \equiv T\frac{dg(T,\kappa)}{dT} = \frac{g}{2\kappa^2}T\frac{d\Pi_T(T,\kappa)}{dT}. \tag{1}$$

Due to the $O(3)$ invariance, the spatial part of the gluon plarization tensor Π_{ij} is expressed as follows:

$$\Pi_{ij}(k) = \Pi_T(\delta_{ij} - \frac{k_i k_j}{\vec{\mathbf{k}}^2}) + \Pi_L\frac{k_i k_j}{\vec{\mathbf{k}}^2} \tag{2}$$

and Π_T can be extracted by applying the projection operator

$$t_{ij} = \frac{1}{2}(\delta_{ij} - \frac{k_i k_j}{\vec{\mathbf{k}}^2}) \tag{3}$$

to Π_{ij}.

The thermal β function has been calculated in BFM at one-loop level for the cases of the gauge parameter $\xi_Q = 0$ [4], $\xi_Q = 1$ [5] and $\xi_Q =$an arbitrary number [2][7]. The results are expressed in a form,

$$\beta_T^{BFM} = \frac{g^3 N}{128}\left\{\frac{7}{16} - \frac{1}{8}(1-\xi_Q) + \frac{1}{64}(1-\xi_Q)^2\right\}\frac{T}{\kappa}, \tag{4}$$

where N is the number of colors. Contrary to the case of the QCD β function at zero temperature, β_T^{BFM} is dependent on the gauge-parameter ξ_Q. The reason

why we have obtained ξ_Q-dependent β_T in BFM is that the contributions to β_T come from the finite part of the gluon polarization tensor $\Pi_{\mu\nu}$ and that BFM gives ξ_Q-dependent finite part for $\Pi_{\mu\nu}$ [8][9].

In this paper I propose to use the pinch technique (PT) to obtain the gauge-independent β_T in a hot Yang-Mills gas. The PT was proposed some time ago by Cornwall [10] for an algorithm to form new gauge-independent proper vertices and new propagators with gauge-independent self-energies. First it was used to obtain the one-loop gauge-independent effective gluon self-energy and vertices in QCD [11] and then it has been applied to the standard model [12].

For example, let us consider the S-matrix element T for the elastic quark-quark scattering at one-loop order. Besides the self-energy diagram in Fig.1, the vertex diagrams of the first kind and the second kind, and the box diagrams, which are shown in Fig.2(a), Fig.3(a), and Fig.4(a), respectively, contribute to T. Such contributions are, in general, gauge-dependent while the sum is gauge-independent. Then we can extract the "pinch parts" of the vertex and box diagrams, which are dipicted in Fig.2(b), Fig.3(b), and Fig.4(b). They emerge when a γ^μ matrix on the quark line is contracted with a four-momentum k_μ offered by a gluon propagator or an elementary three-gluon vertex. Such a term triggers an elementary Ward identity of the form

$$\not{k} = (\not{p} + \not{k} - m) - (\not{p} - m). \tag{5}$$

The first term removes (pinches out) the internal fermion propagator, whereas the second term vanishes on shell, or *vice versa* . This leads to contributions to T with one less fermion propagator and, hence, these contributions are called "pinch parts". The contribution of the self-energy diagram, when added pinch parts from the vertex and box diagrams, become gauge-independent. In this way we can construct the gauge-independent effective gluon polarization tensor (self-energy).

As in the case of BFM, the effective gluon polarization tensor and vertices constructed by PT obey the naive QED-like Ward identities. Thus we can use the same Eq.(1) to obtain β_T in the framework of PT. More importantly, PT gives the gauge-independent results *up to the finite terms*, since they are constructed from S-matrix. It was shown recently [13] that BFM with the gauge parameter $\xi_Q = 1$ reproduces

the PT results at one-loop order. However, for $\xi_Q \neq 1$, this coincidence does not hold any more. In fact, BFM gives at one-loop order the gluon polarization tensor whose finite part is ξ_Q-dependent. Interestingly enough, Papavassiliou [8] showed that when PT is applied to BFM for $\xi_Q \neq 1$ to construct the effective gluon polarization tensor, the gauge dependence of the finite part disappears and the previous $\xi_Q = 1$ result (or the universal PT result) is obtained.

To my knowledge, there exists, so far, only one approach, i.e. PT, which gives the gauge-independent gluon polarization tensor *up to finite terms*. And these finite terms give contributions to β_T. This notion inspires the use of PT for the calculations of β_T. In the following I will show that we obtain the same β_T in the framework of PT even when we calculate in four different gauges, (i) the background field method with an arbitrary gauge, (ii) the Feynman gauge, (iii) the Coulomb gauge, and (iv) the temporal axial gauge.

(i) *The Background Field Method*

In the background field method with an arbitrary gauge, the gluon propagator, $iD^{\mu\nu}_{ab(BFM)} = -i\delta_{ab} D^{\mu\nu}_{(BFM)}$, and the three-gluon vertex with one background gluon field, $\widetilde{\Gamma}^{abc}_{\lambda\mu\nu}$, are given, respectively, as follows [14]:

$$D^{\mu\nu}_{(BFM)} = \frac{1}{k^2}\Big[g^{\mu\nu} - (1-\xi_Q)\frac{k^\mu k^\nu}{k^2}\Big], \tag{6}$$

and

$$\widetilde{\Gamma}^{abc}_{\lambda\mu\nu}(p,k,q) = g f^{bac}\Big[(1-\frac{1}{\xi_Q})\Gamma^P_{\lambda\mu\nu}(p,k,q) + \Gamma^F_{\lambda\mu\nu}(p,k,q)\Big], \tag{7}$$

where

$$\begin{aligned} \Gamma^P_{\lambda\mu\nu}(p,k,q) &= p_\lambda g_{\mu\nu} - q_\nu g_{\lambda\mu} \\ \Gamma^F_{\lambda\mu\nu}(p,k,q) &= 2k_\lambda g_{\mu\nu} - 2k_\nu g_{\lambda\mu} - (2p+k)_\mu g_{\lambda\nu}. \end{aligned} \tag{8}$$

In the vertex, k_μ is taken to be the momentum of the background field and each momentum flows inward and, thus, $p + k + q = 0$.

A one-loop calculation of the polarization tensor was performed in Ref. [7], from which the transverse function $\Pi^{(BFG)}_T(k_0 = 0, \kappa = |\vec{\mathbf{k}}|)$ in the static limit can be

extracted for $\kappa << T$ as follows:

$$\Pi_T^{(BFM)}(T,\kappa) = Ng^2\kappa T\left\{\frac{7}{16} - \frac{1}{8}(1-\xi_Q) + \frac{1}{64}(1-\xi_Q)^2\right\} + \mathcal{O}(\kappa^2). \tag{9}$$

Using this expression for Π_T in Eq.(1), Elmfors and Kobes obtained Eq.(4) for β_T^{BFM} which is indeed gauge-parameter ξ_Q dependent [2].

Now we evaluate the pinch contributions to Π_T. We consider the quark-quark scattering at one-loop order in the Minkowski space. We use the gluon propagator and the three-gluon vertex given in Eqs.(6)-(8). The pinch contributions come from the vertex diagrams of the first kind [Fig. 2(b) and its mirror graph], the vertex diagrams of the second kind [Fig. 3(b) and its mirror graph] and the box-diagrams [Fig. 4(b)]. We can extract from them the pinch contribution to the polarization tensor, which is expressed as [8]

$$\begin{aligned} i\Pi^{\mu\nu}_{P(BFM)} &= Ng^2(1-\xi_Q)k^2\int\frac{d^4p}{(2\pi)^4}\frac{-2kp}{p^4q^2}g^{\mu\nu} \\ &\quad +\frac{N}{2}g^2(1-\xi_Q)^2k^4\int\frac{d^4p}{(2\pi)^4}\frac{-p^\mu p^\nu}{p^4q^4}. \end{aligned} \tag{10}$$

where it is understood that the loop variables are related by $k+p+q=0$.

When we turn to the imaginary time finite temperature formulation, we replace the integral in the Minkowski space with the following one:

$$\int\frac{d^4p}{(2\pi)^4} \Rightarrow i\int\frac{d^3p}{8\pi^3}T\sum_n \tag{11}$$

where the summation goes over the n in $p_0 = 2\pi inT$. Applying the projection operator t_{ij} to the spatial part of $\Pi^{\mu\nu}_{P(BFG)}(k_0=0,\kappa=|\vec{\mathbf{k}}|)$, we obtain in the limit $\kappa << T$,

$$\begin{aligned} \Pi_T^{P(BFM)}(T,\kappa) &= Ng^2(1-\xi_Q)\kappa^2\int\frac{d^3p}{8\pi^3}T\sum_n\frac{2\vec{\mathbf{k}}\cdot\vec{\mathbf{p}}}{p^4q^2} \\ &\quad -\frac{N}{4}g^2(1-\xi_Q)^2\kappa^2\int\frac{d^3p}{8\pi^3}T\sum_n\frac{\vec{\mathbf{k}}^2\vec{\mathbf{p}}^2-(\vec{\mathbf{k}}.\vec{\mathbf{p}})^2}{p^4q^4} \qquad (12) \\ &= Ng^2\kappa T\left\{\frac{1}{8}(1-\xi_Q) - \frac{1}{64}(1-\xi_Q)^2\right\} + \mathcal{O}(\kappa^2). \qquad (13) \end{aligned}$$

Adding two contributions we find that the sum

$$\begin{aligned}\Pi_T(T,\kappa) &= \Pi_T^{(BFM)}(T,\kappa)+\Pi_T^{P(BFM)}(T,\kappa)\\ &= Ng^2\kappa T\frac{7}{16}+\mathcal{O}(\kappa^2)\end{aligned}\tag{14}$$

is gauge-parameter ξ_Q independent, and this gives a gauge-independent thermmal β function:

$$\beta_T=\frac{g^3N}{128}\frac{7}{16}\frac{T}{\kappa}.\tag{15}$$

Note that the result coincides with β_T^{BFM} in Eq.(4) with $\xi_Q=1$ [5].

(ii) *The Feynman Gauge (The Covariant Gauge with $\xi=1$)*

In the Feynman gauge (FG) the gluon propagator, $iD^{\mu\nu}_{ab(FG)}=-i\delta_{ab}D^{\mu\nu}_{(FG)}$, is in a very simple form:

$$D^{\mu\nu}_{(FG)}=\frac{1}{k^2}g^{\mu\nu}.\tag{16}$$

The three-gluon vertex is expressed as

$$\Gamma^{abc}_{\lambda\mu\nu}(p,k,q)=gf^{bac}\Big[\Gamma^P_{\lambda\mu\nu}(p,k,q)+\Gamma^F_{\lambda\mu\nu}(p,k,q)\Big]\tag{17}$$

where $\Gamma^P_{\lambda\mu\nu}(p,k,q)$ and $\Gamma^F_{\lambda\mu\nu}(p,k,q)$ are given in Eq.(8). From the one-loop polarization tensor in FG, we extract the contribution to the transverse function $\Pi_T^{(FG)}$ and find in the limit $\kappa << T$,

$$\Pi_T^{(FG)}(T,\kappa)=Ng^2\kappa T\frac{3}{16}+\mathcal{O}(\kappa^2).\tag{18}$$

The pinch contribution to the polarization tensor in FG is very simple. Since the gluon propagator in FG does not have a $k^\mu k^\nu$ term, the only contribution is coming from the vertex diagram of the second kind with the three-gluon vertex Γ^P (and its mirror graph) [11], and it is given by

$$i\Pi^{\mu\nu}_{P(FG)}=2Ng^2k^2\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2q^2}g^{\mu\nu}.\tag{19}$$

Following the same procedure as we did in (i), we obtain for the pinch contribution to Π_T^{FG} as

$$\Pi_T^{P(FG)}(T,\kappa)=Ng^2\kappa T\frac{1}{4}+\mathcal{O}(\kappa^2).\tag{20}$$

Again when we add two contributions, $\Pi_T^{(FG)}$ and $\Pi_T^{P(FG)}$, we obtain the same Π_T in Eq.(14) and, thus, the same β_T in Eq.(15).

(iii) *The Coulomb Gauge*

In the frame of a unit vector $n^\mu = (1,0,0,0)$, the gluon propagator in the Coulomb gauge (CG), $iD^{\mu\nu}_{ab(CG)} = -i\delta_{ab}D^{\mu\nu}_{(CG)}$, is defined by

$$D^{\mu\nu}_{(CG)} = \frac{P^{\mu\nu}}{k^2} + \frac{1}{k^2}\Big[Q^{\mu\nu} + \frac{\sqrt{2}k_0}{|\vec{\mathbf{k}}|}C^{\mu\nu} - \frac{k_0^2}{\vec{\mathbf{k}}^2}D^{\mu\nu}\Big] \tag{21}$$

where

$$\begin{aligned} P^{\mu\nu}(k) &= g^{\mu\nu} - n^\mu n^\nu + \frac{1}{\vec{\mathbf{k}}^2}\Big[k^\mu k^\nu - (n^\mu k^\nu + k^\mu n^\nu)k_0 + n^\mu n^\nu k_0^2\Big], & (22)\\ Q^{\mu\nu}(k) &= -\frac{k^2}{\vec{\mathbf{k}}^2}n^\mu n^\nu + \frac{k_0}{\vec{\mathbf{k}}^2}\Big[n^\mu k^\nu + k^\mu n^\nu\Big] - \frac{k^\mu k^\nu k_0^2}{\vec{\mathbf{k}}^2 k^2}, & (23)\\ C^{\mu\nu}(k) &= -\frac{1}{\sqrt{2}|\vec{\mathbf{k}}|}(n^\mu k^\nu + k^\mu n^\nu) + \frac{\sqrt{2}k_0}{|\vec{\mathbf{k}}|k^2}k^\mu k^\nu, & (24)\\ D^{\mu\nu}(k) &= \frac{k^\mu k^\nu}{k^2}. & (25) \end{aligned}$$

The three-gluon vertex is the same as in FG, that is, $\Gamma^{abc}_{\lambda\mu\nu}(p,k,q)$ in Eq.(17).

The transverse function Π_T is related to the polarization tensor as

$$\Pi_T = t_{ij}\Pi_{ij} = \frac{1}{2}\Big[\Pi_{ii} - \frac{1}{\vec{\mathbf{k}}^2}k_i\Pi_{ij}k_j\Big]. \tag{26}$$

Since $k_i\Pi_{ij}k_j = 0$ in the static limit $k_0 = 0$, we have $\Pi_T(T,\kappa) = \frac{1}{2}\Pi_{ii}(k_0 = 0, \kappa)$. The Π_{ii} for general k_0 and κ was evaluated in CG and the temporal axial gauge in Ref. [15]. Using the expression of Eq.(4.38) in Ref. [15] for $\Pi_{ii}^{CG}(k_0,\kappa)$, we can calculate the static limit of Π_{ii}^{CG} and obtain

$$\Pi_T^{(CG)}(T,\kappa) = \frac{1}{2}\Pi_{ii}^{(CG)}(0,\kappa) = Ng^2\kappa T\frac{9}{64} + \mathcal{O}(\kappa^2). \tag{27}$$

It is straightforward to calculate the pinch contributions to the polarization tensor in CG. Again we consider the quark-quark scattering at one-loop order. We discard the terms which are proportional to $\not{n}$ and $\not{k}$, where k_μ is the momentum transfered in the scattering, and we make use of the Dirac equations satisfied by the

external quark fields, such as, $(\not p - m)u(p) = 0$ and $\bar{u}(p)(\not p - m) = 0$. The pinch contribution to the polarization tensor in CG is expressed as

$$
\begin{aligned}
i\Pi^{\mu\nu}_{P(CG)} &= Ng^2k^2\int\frac{d^4p}{(2\pi)^4}\Big[\frac{k^2}{p^2q^2\vec{\mathbf{p}}^2} - \frac{1}{p^2\vec{\mathbf{p}}^2} - \frac{4\vec{\mathbf{k}}\cdot\vec{\mathbf{p}}}{p^2q^2\vec{\mathbf{p}}^2}\Big]g^{\mu\nu} \\
&\quad + \frac{N}{2}g^2k^2\int\frac{d^4p}{(2\pi)^4}\frac{p_\mu p_\nu}{p^2q^2}\Big[\frac{k^2}{\vec{\mathbf{p}}^2\vec{\mathbf{q}}^2} - \frac{4}{\vec{\mathbf{p}}^2} + \frac{2\vec{\mathbf{k}}^2}{\vec{\mathbf{p}}^2\vec{\mathbf{q}}^2}\Big]. \qquad (28)
\end{aligned}
$$

Applying the projection operator t_{ij} to the spatial part $\Pi^{ij}_{P(CG)}$, we obtain in the static limit and for $\kappa << T$

$$
\begin{aligned}
\Pi^{P(CG)}_T(T,\kappa) &= -Ng^2\kappa^2\int\frac{d^3p}{8\pi^3}T\sum_n\Big\{\frac{1}{q^2\vec{\mathbf{p}}^2} + \frac{2\vec{\mathbf{k}}\cdot\vec{\mathbf{p}}}{p^2q^2\vec{\mathbf{p}}^2}\Big\} \\
&\quad -\frac{N}{4}g^2\kappa^2\int\frac{d^3p}{8\pi^3}T\sum_n\Big[1 - \frac{(\vec{\mathbf{k}}\cdot\vec{\mathbf{p}})^2}{\vec{\mathbf{k}}^2\vec{\mathbf{p}}^2}\Big]\Big\{\frac{\vec{\mathbf{k}}^2}{p^2q^2\vec{\mathbf{q}}^2} - \frac{4}{p^2q^2}\Big\} \qquad (29) \\
&= Ng^2\kappa T\frac{19}{64} + \mathcal{O}(\kappa^2). \qquad (30)
\end{aligned}
$$

Adding the two contributions, $\Pi^{(CG)}_T$ and $\Pi^{P(CG)}_T$, we find that the sum is equal to Π_T in Eq.(14) and, thus, we obtain the same β_T in Eq.(15).

(iv) *The Temporal Axial Gauge*

The gluon propagator in the temporal axial gauge (TAG), $iD^{\mu\nu}_{ab(TAG)} = -i\delta_{ab}D^{\mu\nu}_{(TAG)}$, is defined by

$$
D^{\mu\nu}_{(TAG)} = \frac{P^{\mu\nu}}{k^2} + \frac{1}{k^2}\Big[Q^{\mu\nu} + \frac{|\vec{\mathbf{k}}|}{\sqrt{2}k_0}C^{\mu\nu} - \frac{\vec{\mathbf{k}}^2}{k_0^2}D^{\mu\nu}\Big] \qquad (31)
$$

where $P^{\mu\nu}$, $Q^{\mu\nu}$, $C^{\mu\nu}$ and $D^{\mu\nu}$ are given in Eqs.(22 -25). The three-gluon vertex is given by $\Gamma^{abc}_{\lambda\mu\nu}(p,k,q)$ in Eq.(17). The static limit of $\Pi^{(TAG)}_{ii}$ was caluculated in Ref. [15]. Using the result of Eq.(4.44) in Ref. [15], we find for $\kappa << T$

$$
\Pi^{(TAG)}_T(T,\kappa) = \frac{1}{2}\Pi^{(TAG)}_{ii}(0,\kappa) = Ng^2\kappa T\frac{5}{16} + \mathcal{O}(\kappa^2). \qquad (32)
$$

Following the same procedure as we extracted the pinch parts from the one-loop quark-quark scattering diagrams in CG, we obtain for the pinch contribution to the polarization tensor in TAG,

$$
i\Pi^{\mu\nu}_{P(TAG)} = Ng^2k^2\int\frac{d^4p}{(2\pi)^4}\Big[\frac{k^2}{p^2q^2p_0^2} - \frac{1}{p^2p_0^2} + \frac{2}{q^2p_0^2} - \frac{4\vec{\mathbf{k}}\cdot\vec{\mathbf{p}}}{p^2q^2p_0^2}\Big]g^{\mu\nu}
$$

$$+\frac{N}{2}g^2k^2\int\frac{d^4p}{(2\pi)^4}\frac{p_\mu p_\nu}{p^2q^2}\Big[\frac{k^2}{p_0^2q_0^2}-\frac{4}{p_0^2}+\frac{2\vec{\mathbf{k}}^2}{p_0^2q_0^2}\Big]. \tag{33}$$

Then in the static limit, $\Pi_T^{P(TAG)}$ is expressed as

$$\begin{aligned}\Pi_T^{P(TAG)}(T,\kappa) &= -Ng^2\kappa^2\int\frac{d^3p}{8\pi^3}T\sum_n\Big\{\frac{\vec{\mathbf{k}}^2+4\vec{\mathbf{k}}\cdot\vec{\mathbf{p}}}{p^2q^2p_0^2}+\frac{1}{p^2p_0^2}-\frac{2}{q^2p_0^2}\Big\}\\ &\quad -\frac{N}{4}g^2\kappa^2\int\frac{d^3p}{8\pi^3}T\sum_n\Big[\vec{\mathbf{p}}^2-\frac{(\vec{\mathbf{k}}\cdot\vec{\mathbf{p}})^2}{\vec{\mathbf{k}}^2}\Big]\Big\{\frac{\vec{\mathbf{k}}^2}{p^2q^2p_0^2q_0^2}-\frac{4}{p^2q^2p_0^2}\Big\}\end{aligned} \tag{34}$$

Due to the $1/p_0^2$ and $1/q_0^2$ terms coming from the TAG propagator, the above integrand contains a $\vec{\mathbf{k}}^2/\vec{\mathbf{p}}^2$ singularity at the lower limit of the integration. This singularity is circumvented by the principal value prescription [15][16]. The result is for $\kappa << T$

$$\Pi_T^{P(TAG)}(T,\kappa) = Ng^2\kappa T\frac{1}{8}+\mathcal{O}(\kappa^2). \tag{35}$$

Again the sum of $\Pi_T^{(TAG)}$ and $\Pi_T^{P(TAG)}$ coincides with Π_T in Eq.(14) and yields the same β_T in Eq.(15).

I have demonstrated the calculation of the thermal β function β_T in four different gauges, that is, in BFM with an arbitrary gauge, in FG, in CG, and in TAG. When the pinch contributions are taken care of, the same result $\beta_T=\frac{g^3N}{128}\frac{7}{16}\frac{T}{\kappa}$ was obtained at one-loop order in all four cases. More details will be reported elsewhere [17]. However, this is not the end of the story. Elmfors and Kobes pointed out [2] that the leading contribution to β_T, which gives a term T/κ, does not come from the hard part of the loop integral, responsible for a T^2/κ^2 term, but from soft loop integral. Hence they emphasized that it is not consistent to stop the calculation at one-loop order for soft internal momenta and that the resummed propagator and vertices [18] must be used to get the complete leading contribution. Since the corrections to the bare propagator and vertices, which come from the hard thermal loops, are gauge-independent and satisfy simple Ward identities [18], it is well-expected that we will obtain the gauge-independent thermal β function even when we use the resummed propagator and vertices in the framework of PT. Study along this direction is under way.

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Figure caption

Fig.1

The self-energy diagram for the quark-quark scattering.

Fig.2

(a) The vertex diagrams of the first kind for the quark-quark scattering. (b) Their pinch contribution.

Fig.3

(a) The vertex diagram of the second kind for the quark-quark scattering. (b) Its pinch contribution.

Fig.4

(a) The box diagrams for the quark-quark scattering. (b) Their pinch contribution.

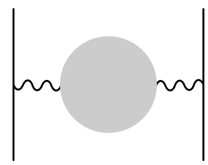

Figure 1

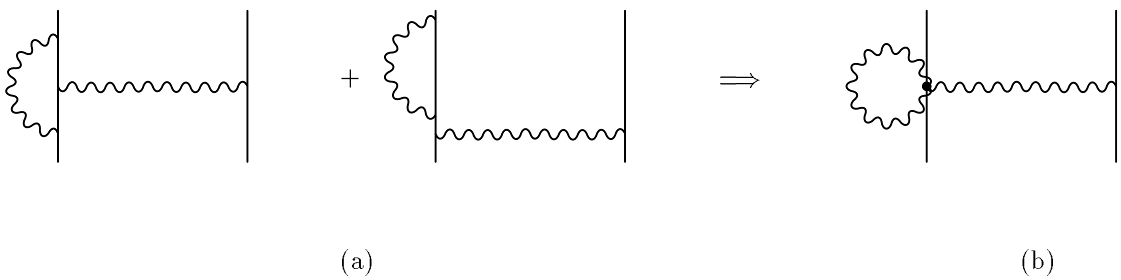

Figure 2

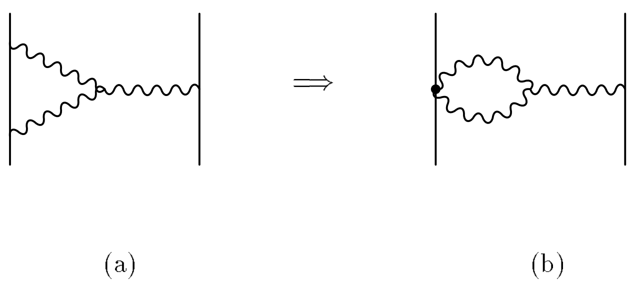

Figure 3

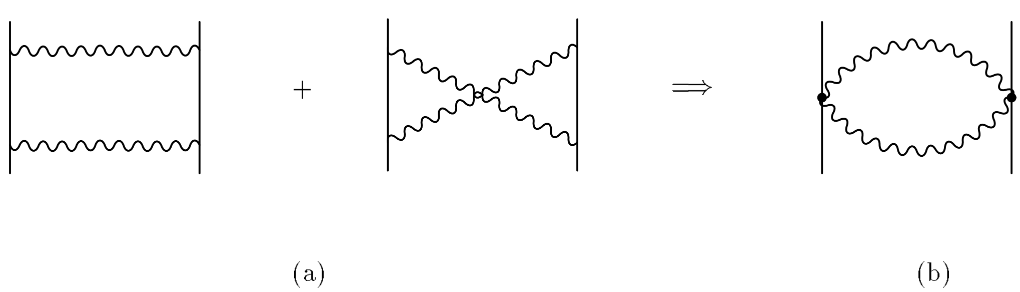

Figure 4